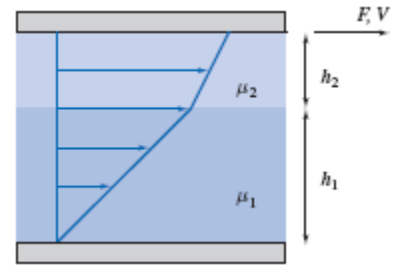


Problem 2.56

[Difficulty: 2]

2.56 Fluids of viscosities $\mu_1 = 0.1 \text{ N} \cdot \text{s}/\text{m}^2$ and $\mu_2 = 0.15 \text{ N} \cdot \text{s}/\text{m}^2$ are contained between two plates (each plate is 1 m^2 in area). The thicknesses are $h_1 = 0.5 \text{ mm}$ and $h_2 = 0.3 \text{ mm}$, respectively. Find the force F to make the upper plate move at a speed of 1 m/s . What is the fluid velocity at the interface between the two fluids?



Given: Flow between two plates

Find: Force to move upper plate; Interface velocity

Solution:

The shear stress is the same throughout (the velocity gradients are linear, and the stresses in the fluid at the interface must be equal and opposite).

Hence
$$\tau = \mu_1 \cdot \frac{du_1}{dy} = \mu_2 \cdot \frac{du_2}{dy} \quad \text{or} \quad \mu_1 \cdot \frac{V_i}{h_1} = \mu_2 \cdot \frac{(V - V_i)}{h_2} \quad \text{where } V_i \text{ is the interface velocity}$$

Solving for the interface velocity V_i

$$V_i = \frac{V}{1 + \frac{\mu_1}{\mu_2} \cdot \frac{h_2}{h_1}} = \frac{1 \cdot \frac{\text{m}}{\text{s}}}{1 + \frac{0.1}{0.15} \cdot \frac{0.3}{0.5}} \quad V_i = 0.714 \frac{\text{m}}{\text{s}}$$

Then the force required is

$$F = \tau \cdot A = \mu_1 \cdot \frac{V_i}{h_1} \cdot A = 0.1 \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2} \times 0.714 \cdot \frac{\text{m}}{\text{s}} \times \frac{1}{0.5 \cdot \text{mm}} \times \frac{1000 \cdot \text{mm}}{1 \cdot \text{m}} \times 1 \cdot \text{m}^2 \quad F = 143 \text{ N}$$